AP Calculus AB – Carlson   Summer Work

Late assignments will be awarded partial credit where appropriate.

Welcome to AP Calculus AB! Over the next year, we're going to have a lot of fun doing some really cool math to learn how the world around us works. You will be carrying on a tradition of exploring calculus that leads back to everyone's favorite physicist (OK, not everyone has a favorite physicist, though they should), Sir Isaac Newton.

To get ready for our course this fall, I'm asking you to complete some preparatory work to help you start off the year on the right foot. This should be mostly (almost entirely) review from Algebra II and Precalculus, and will cover: General Algebra, Linear Functions, Functions, Logarithms and Exponential Functions, Trigonometry, and using your calculator.

I've worked hard to focus the summer work on only problems that help you get ready for calculus in the fall, and each year have improved the summer work assignment based on student feedback, and figuring out which problems worked well, and which were a waste of time, so I sincerely hope you find the summer work experience to be somewhat less than pure torture.

Additionally—**Mr. Butterworth and I will be offering two sessions this summer (August 18, and TBD)** to invite any students who would like to spend a little time doing some summer work with support, or helping shore up some areas you might be weak on. This is a new experiment, and one I hope will lead to a better experience for anyone who is able to come in.

Feel free to email me at any time, if you have questions on the assignment, or about the year to come.

Enjoy the summer,

Mr. Carlson

Summer Assignment Part 1:

Due July 31, 11:59:59 PM EST. 10 points

Please visit: http://www.tofercarlson.com/calculus

Note: The website will be updated during the week of final exams, so please wait until after your exams are completed to explore our website, otherwise it will have last year's information.

Click on the summer work link and use the form to send me an email introducing yourself. Respond to the questions listed on the website.

Summer Assignment Part 2.

Due at the start of our first class meeting (Day 1 or 2 of the school year, depending on schedule). 60 points (note: quizzes tend to be worth 40 points) For any credit, all work must be shown.

I. General Algebra:

Concepts: intercepts, intersections, solving equations, domain and range, general shapes of families of graphs.

Intercepts are defined as places that our graph crosses the x-axis or y-axis.
To find x-intercepts: set $x = 0$, and solve for y. To find y-intercepts: set $y = 0$, and solve for x.

Try it:
1. In each equation, find the intercepts (x and y), if any.

a. $y = \frac{5}{3}x - 8$

b. $y = (2x-3)(x-7)$

c. $y = \frac{2x+5}{x-2}$

d. $xy = 12$

Intersections occur when two equations share a solution. When you began solving systems of equations, you found intersections graphically, and then using algebraic methods (substitution and elimination).

2. In each of these cases, use algebra (substitution, or some other method) to find the intersections of the equations without graphing.

a. $y = 4x - 7$
$2x + 3y = 12$

b. $y = \frac{3x}{4}$
$x^2 + y^2 = 15$

Solving equations means finding a set of values that make each equation true. Often, there are lots of solutions to an equation. Sometimes, there are no solution to an equation. You should be able to solve a variety of equations, including linear, quadratic, polynomial, exponential, logarithmic, trigonometric, radical, and rational equations. Whew! That was a lot to write.

3. Find all possible solutions for each equation.

a. $3(x-7)+13=4x-14$

b. $3x^2-7x=4$

c. $\sqrt[4]{x-5}+3=7$

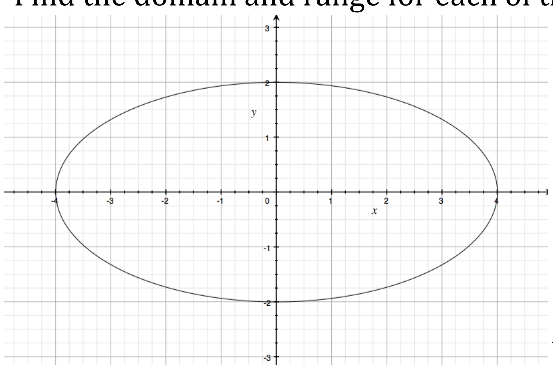
d. $\dfrac{2x+6}{x-2}+1=x$

Domain and range – you should be able to find domain (all possible x-values for a relation) and range (all possible y-values for a relation) for a variety of functions algebraically. We will discuss this further in the fall, so don't freak out. What I'd really like you do now, is find domain and range given a graph.

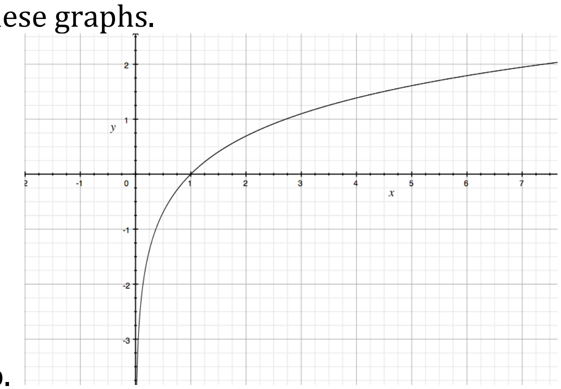
Domain: Identify where on the coordinate plane that our graph has an x-value. If the graph appears to go off the grid that we can see, we can assume (for the sake of these questions) that it will continue in about the same direction forever.

Range: Identify where on the coordinate plane that our graph has a y-value. If the graph appears to go off the grid that we can see, we can assume (for the sake of these questions) that it will continue in about the same direction forever.

4. Find the domain and range for each of these graphs.



a.
Domain:
Range:



b.
Domain:
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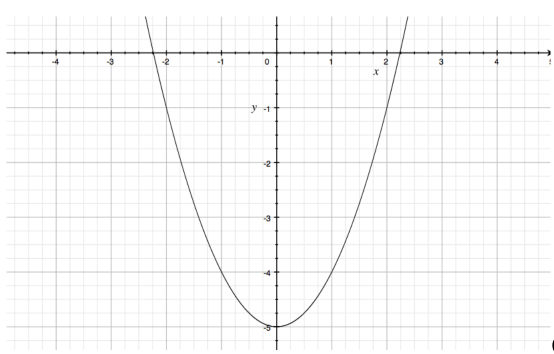
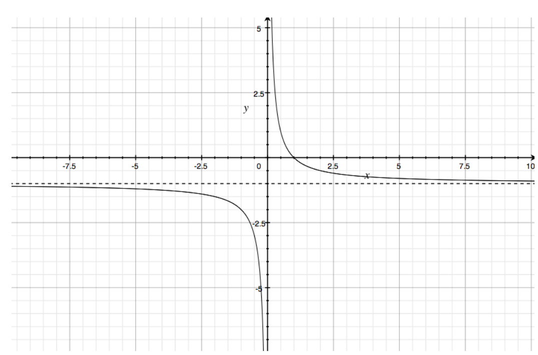
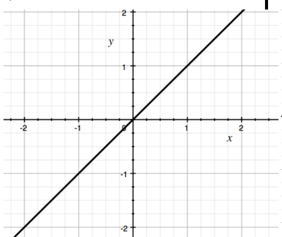
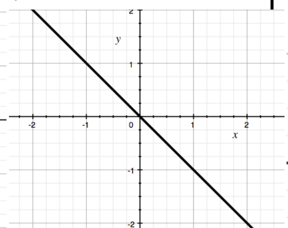
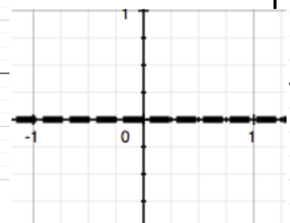
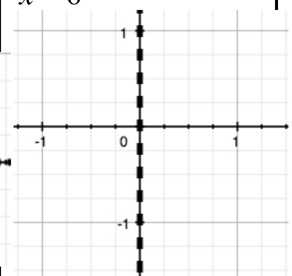
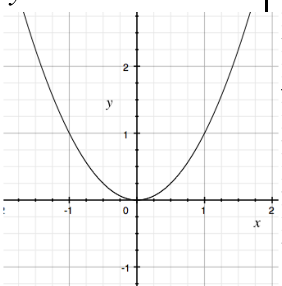
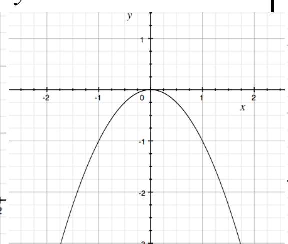
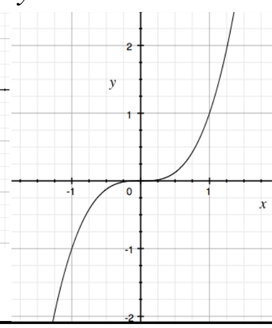
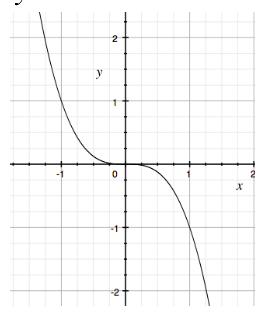
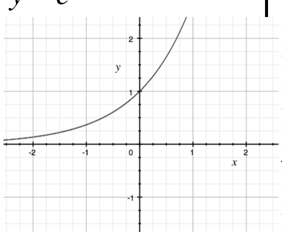
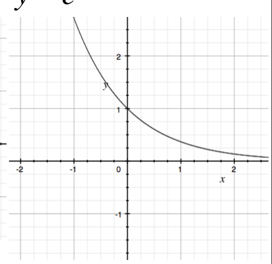
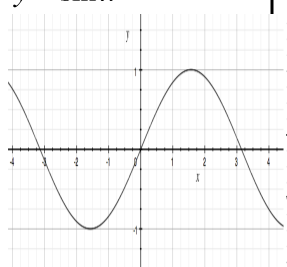
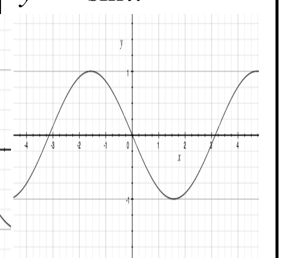
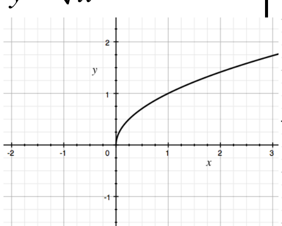
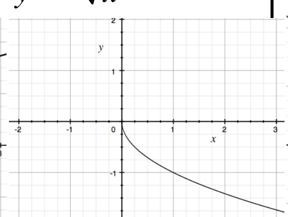
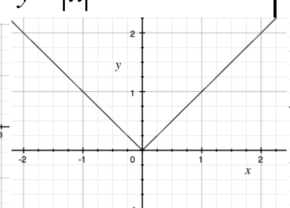
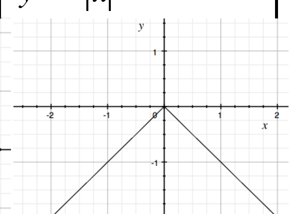
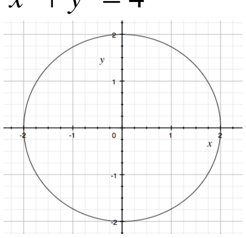
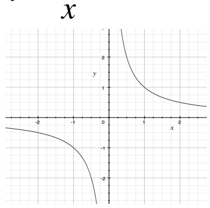
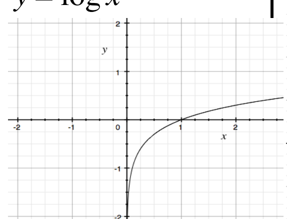
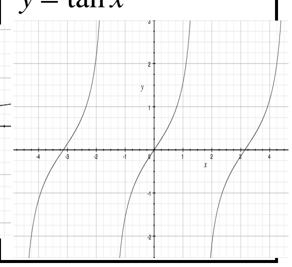


c.
Domain:
Range:



d.
Domain:
Range:

Shapes of General Graphs – by this point in your high school career, you've spent a great deal of time graphing equations. There are certain graphs that I would like you to be able to know the general shape of. Here's a list, with their graphs—they should already be familiar to you. For any you don't know, it might be worth making flash cards to help you get them under your belt.

$y=x$ 	$y=-x$ 	$y=0$ 	$x=0$ 
$y=x^2$ 	$y=-x^2$ 	$y=x^3$ 	$y=-x^3$ 
$y=e^x$ 	$y=e^{-x}$ 	$y=\sin x$ 	$y=-\sin x$ 
$y=\sqrt{x}$ 	$y=-\sqrt{x}$ 	$y=\lvert x\rvert$ 	$y=-\lvert x\rvert$ 
$x^2+y^2=4$ 	$y=\frac{1}{x}$ 	$y=\log x$ 	$y=\tan x$ 

Additionally, you should know what happens if you change an equation by performing transformations.

4 (continued). Describe in words the transformation each graph undergoes as the equation changes.

e. $f(x)=\sin x \rightarrow 2f(x-\pi)=2\sin(x-\pi)$	
f. $g(x)=\sqrt{x} \rightarrow -g(x-2)+3=-\sqrt{x-2}+3$	
g. $h(x)=x^2-4 \rightarrow \lvert h(x)\rvert=\lvert x^2-4\rvert$	

II. Linear Functions.

Concepts: finding slopes and average rates of change, interval notation, writing equations for lines in point-slope form of an equation for a line.

5. Find the slope between the indicated points.

a. $(-\frac{3}{5},13)$ and $(-17,10)$

b. $(3\sqrt{2},3)$ and $(2,5)$

Point Slope Form of an equation for a line is exceptionally useful in calculus—we will spending a lot of time discussing writing equations for lines, and in almost all cases, we will write them using point-slope form:

$$y - y_1 = m(x - x_1) \text{ or } y = y_1 + m(x - x_1)$$

6. Write an equation for (a) the vertical line and (b) the horizontal line through the point P. Write an equation for (c) a line through point P, with slope equal to 1/7. $P(3,2)$	(a) (b) (c)
7. Write an equation for (a) the vertical line and (b) the horizontal line through the point P. Write an equation for (c) a line through point P, with slope equal to $\sqrt{5}$. $P(0,-\sqrt{2})$	(a) (b) (c)
8. Write the point-slope equation for the line through the point P with slope *m*. a) $P(0,3), m = 2$ b) $P(-4,7), m = -2/3$	(a) (b)
9. Which of the following are equations of the line through (-3,4) with slope ½?	(a) y-4=1/2(x+3) (b) y+3=1/2(x-4) (c) y-4=-2(x+3) (d) y-4=2(x+3) (e) y+3=2(x-4) (f) y=4+1/2(x+3) Explain your reasoning.

A note on ***interval notation*** – we can use interval notation to express a range of numbers, much in the same way we use inequalities. Here's a set of intervals with their corresponding inequality notations to help you suss[1] this out.

Interval	Inequality
$[2,20)$	$2 \le x < 20$
$(-\infty,2)$	$-\infty < x < 2$
$[a,b]$	$a \le x \le b$
$(c,12]$	$c < x \le 2$

10. Complete the table with interval notation or inequalities.

Interval	Inequality
$[3,100)$	a.
b.	$3 < x < \infty$
$(2,6]$	c.
d.	$e \le x \le \pi$

The ***average rate of change*** of a function on an interval is defined as the slope between two points on a function.

For example:
Given the function: y = x^2+2, find the average rate of change on the interval [0,4] (between x = 0 and x = 4).

1. Find the points with x-coordinates 0 and 4.
    a. Plug in x = 0;
    b. y = 0 + 2 = 2;
    c. (0,2) is our first point.
    d. Plug in x = 4;
    e. y = 4^2 + 2 = 18;
    f. (4,18) is our second point.
2. Find the slope between the points: (0,2) to (4,18) has a slope of 4.

The average rate of change of y = x^2+2 on [0,4] is 4.

11. Find the average rate of change for each function on the given interval.

a. $y = 2x^2$, on the interval $[-1,3]$

b. $y = \ln(x^3)$, on the interval $[1,e]$

[1] http://www.merriam-webster.com/dictionary/suss

III. Functions

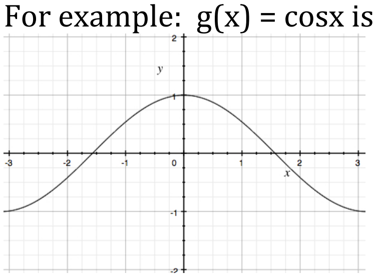
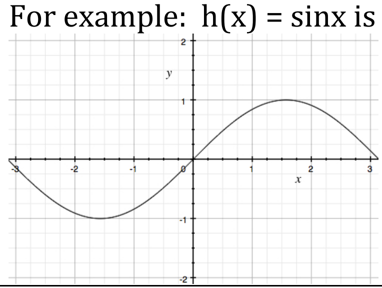
Concepts: function notation, function composition and operations, even and odd functions, inverse functions, piecewise functions.

We will be using function notation throughout this course, and write functions both in their algebraic form, and their function notation. For instance, if we let $f(x)=3\sin^2 x$, we could refer to the function as: $3\sin^2x$, f(x), or f.

You should also be able to perform function operations and compositions as indicated by function notation.

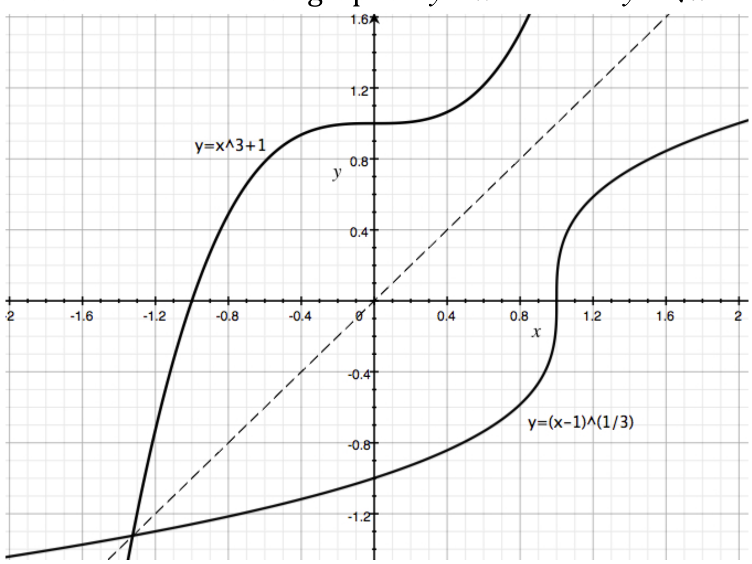
12. Given $f(x)=x+5$, $g(x)=x^2-3$ Find (a) f(g(x)) (b) g(f(x)) (c) f(g(0)) (d) g(f(0)) (e) g(g(-2)) (f) f(f(x)) (g) 2f(x)+4	(a) (b) (c) (d) (e) (f) (g)

You should be able to determine algebraically and graphically whether a function is even or odd.

Even Functions	Odd Functions
Algebraically: f(-x)=f(x)… in other words, if you plug in –x for all x in the function, you'll get the same function back. *Geometrically*: A function that is even, is symmetric about the y-axis. For example: g(x) = cosx is even. 	*Algebraically*: f(-x)=-f(x)… in other words, if you plug in –x for all x in the function, you'll get the opposite function back. *Geometrically*: A function that is odd, is symmetric about the origin. For example: h(x) = sinx is odd 
Important: Functions can be neither even, nor odd if they don't fit these characteristics.	

13. Determine whether the function is even, odd, or neither. Justify your answer. a) $y = x^4$ b) $y = \sqrt{x^2 + 2}$ c) $y = \dfrac{1}{x-1}$ d) y = sin(2x)	a) b) c) d)

Inverse Functions – sometimes a pair of particular functions will undo each other. We call these functions inverse – and can describe this using function notation: $f^{-1}(f(x)) = x$ (that is, if you plug an inverse into its function, the result is x). Geometrically, two functions that are inverses are reflections over the line y = x. Take a look at these inverse function graphs: $y = x^3 + 1$ and $y = \sqrt[3]{x-1}$



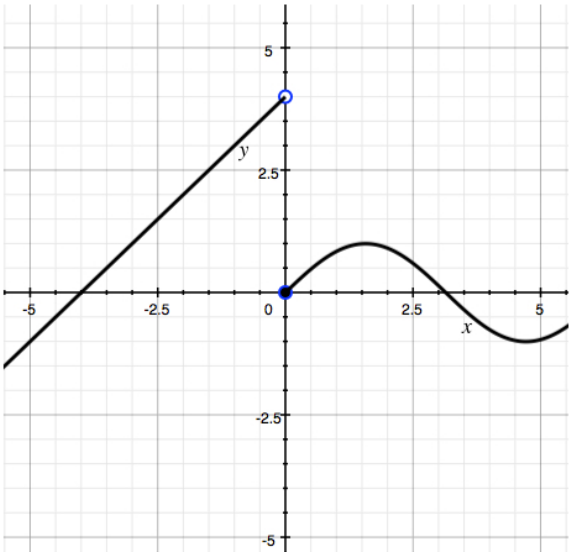
Only some functions have inverses that are also functions – these are *one-to-one* functions, (each element of the domain has exactly one paired element in the range, and each element of the range has exactly one paired element in the domain). When you previously looked at these, you often used the *horizontal line test*—if any horizontal line drawn on a graph of the function crosses the function more than once, it does not have an inverse.

If a function is invertible (if a function has an inverse), you can find its inverse using some cool algebra: switch your x and y, and solve for the new y. Generally, if you've avoided having to use a "±" or inverse trig at this point, your result will still be a function.

14. Determine whether the function $y=\frac{3}{x-2}-1$ has an inverse function. If so, identify the inverse function. Justify your answer.	
15. Determine whether the function $y=x^2-4x+6$ has an inverse function. If so, identify the inverse function. Justify your answer.	
16. Determine whether the function $y=\ln(x-7)$ has an inverse function. If so, identify the inverse function. Justify your answer.	
17. Given: f(x)=2x+3, find $f^{-1}(x)$, and verify that f(x) and f^{-1}(x) are inverses algebraically.	
18. Show that each function f is its own inverse.	$(a) f(x)=\sqrt{1-x^2}, x\geq 0$ $(b) f(x)=1/x$

Piecewise functions – Some functions are made up of more than one part of a function, we graph each part of these separately, and combine them on a single graph.

For example, the graph of: $f(x) = \begin{cases} x+4, x<0 \\ \sin x, x \geq 0 \end{cases}$ can be seen below:

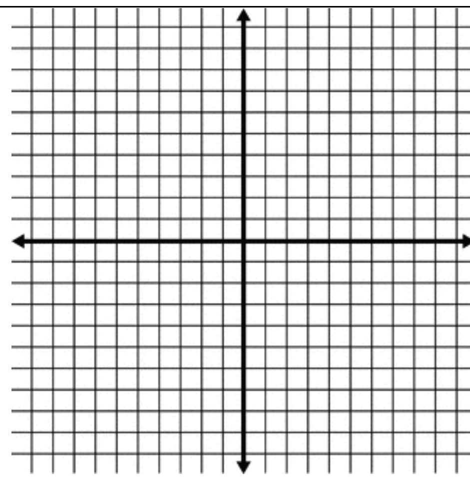


Note that the graph of f(x) = x+4 stops at the y-axis, and the graph of f(x)=sinx starts at the x-axis and continues to the right. Also note that we use an open circle to denote a non-included endpoint (x<0), and a closed circle to denote an included endpoint (x≥0).

For some more help check out: http://www.coolmath.com/algebra/21-advanced-graphing/03-piecewise-functions-01.htm

19. Graph the piecewise-defined functions.

$$f(x) = \begin{cases} 3-x, & x \leq 1 \\ 2x, & 1 < x \end{cases}$$



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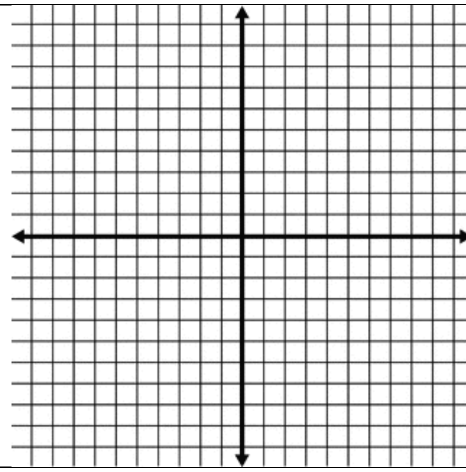
<td>20. Graph the piecewise-defined functions.
$$f(x)=\begin{cases} x^2, x<0 \\ x^3, 0\le x\le 2 \\ 2x-1, x>2 \end{cases}$$</td>
<td></td>
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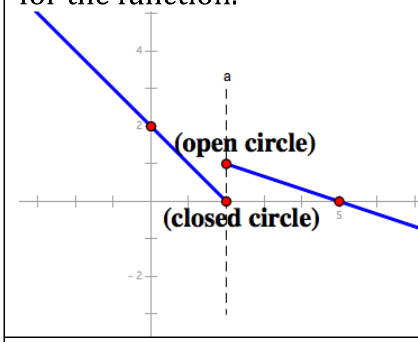
<td>12. Write a piecewise formula for the function.
</td>
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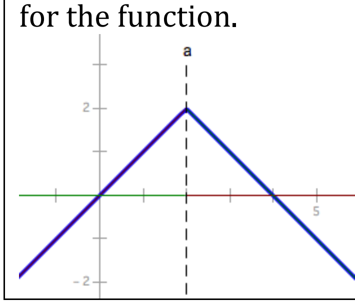
<td>13. Write a piecewise formula for the function.
</td>
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IV. Exponential and Logarithmic functions

Content: convert exponential equations into logarithmic equations and vice versa, use properties of exponents and logarithms to expand or condense expressions, evaluate logarithms for values without a calculator, solve exponential equations using logarithms, solve logarithmic equations using their exponential inverse operation.

13. Convert each logarithmic equation into an equivalent exponential equation: (a) $2 = \log_3 9$ (b) $\log_\pi 17 = 4x + 1$ (c) $\ln c = x$	14. Evaluate each logarithmic expression without a calculator. (a) $\log_3 \sqrt{27}$ (b) $\log(100^{3x+4})$ (c) $\ln 1$
15. Condense each logarithmic expression into a single term. (a) $3 + 2\log_2 x + \frac{1}{2}\log_2 y - \log_2 z$ (b) $\frac{1}{2}(\log_2 x - 2(\log_2 y - 3\log_2 z))$	16. Condense each expontial expression into a single base with one exponent. HINT: (a^{3x+2} not $a^{3x}a^2$) (a) $(3^{x+2})(3^x)(3^{-3x+1})$ (b) $\frac{(e^{2x-4})(e^{x+1})^x}{e}$ Expand the exponential expression so that each exponent has only one term. HINT: ($a^{3x}a^2$ not a^{3x+2}) (c) $e^{.17x+C}$

Solve each logarithmic or exponential equation.

17. Solve the equation algebraically. Show all work. $(1.045)^t = 2$	18. Solve the equation algebraically. Show all work. $3e^{2t-1} + 7 = 12$

19. Solve for y: $\ln y = 2t + 4$	20. Solve for y: $2\log(y+5) - 4 = 2x$

V. Trigonometry

Content: Calculate values of sine, cosine, and tangent for angles with reference angles that are special right triangles; given a trig value, find other trig values of the same angle; solve equations involving trigonometric expressions; use radians fluently (AP Calculus doesn't use degrees); be able to use some trig identities (Pythagorean, quotient, reciprocal).

21. Find exact values of each trig expression without using a calculator.

(a) $\sin\frac{\pi}{4}$	(b) $\cos-\frac{7\pi}{6}$	(c) $\tan 2\pi$	(d) $\cos 0$	(e) $\cos\frac{5\pi}{4}$	(f) $\tan\frac{4\pi}{3}$
(g) $\sin -4\pi$	(h) $\tan\frac{7\pi}{2}$	(i) $\cos\frac{5\pi}{6}$	(j) $\cos\frac{7\pi}{6}$	(k) $\sin\frac{2\pi}{3}$	(l) $\tan\frac{\pi}{4}$

22. Evaluate each expression without a calculator using radians.

(a) $\sin^{-1}(\frac{\sqrt{3}}{2})$	(b) $\tan^{-1}(-1)$	(c) $\sin^{-1}(-1)$	(d) $\tan^{-1}(\sqrt{3})$

23. $\cos\theta = -\frac{15}{17}, \quad \sin\theta > 0$	Find all the trigonometric values of **θ** in radians with the given conditions. $\sin\theta$ $\tan\theta$ $\sec\theta$ $\cot\theta$ $\csc\theta$
24. $\tan\theta = -1, \quad \sin\theta < 0$	Find all the trigonometric values of **θ** with the given conditions. $\sin\theta$ $\tan\theta$ $\sec\theta$ $\cot\theta$ $\csc\theta$

Trigonometry Equations: Solve each equation for all possible values of x.

25. $\tan^2 x - 3 = 0,\ 0 \le x \le 2\pi$	26. csc x=2, $0 < x < 2\pi$
27. $\sin^2 x - 2\sin x = 0$	28. $-\cos^2 x = 2\sin x + 2$

Trig Identities. We've all got 'em, we all need 'em, what do we do with them? You should be able to perform substitution using these identities to simplify problems. This can make it easier to solve some equations, and in calculus, we'll be using them to prove that some of the math we do works. Generally, these are the ones you should at least recall existing by the time you start next year.

Identities		
Pythagorean: Can all be derived from the first: 1. $\sin^2 x + \cos^2 x = 1$ (also $\sin^2 x = 1 - \cos^2 x$ and $\cos^2 x = 1 - \sin^2 x$) 2. $\tan^2 x + 1 = \sec^2 x$ 3. $1 + \cot^2 x = \csc^2 x$	Quotient and Reciprocal: 1. $\sin\theta = \frac{1}{\csc\theta}, \csc\theta = \frac{1}{\sin\theta}$ 2. $\cos\theta = \frac{1}{\sec\theta}, \sec\theta = \frac{1}{\cos\theta}$ 3. $\tan\theta = \frac{1}{\cot\theta}, \cot\theta = \frac{1}{\tan\theta}$ 4. $\tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}$	Sum/Double Angles: 1. $\sin 2\theta = 2\sin\theta\cos\theta$ 2. $\cos 2\theta = \cos^2\theta - \sin^2\theta$ 3. $\sin(a \pm b) = \sin a\cos b \pm \cos a\sin b$ 4. $\cos(a \pm b) = \cos a\cos b \mp \sin a\sin b$

Use trigonometric Identities to rewrite each expression as a single term.

29. $\sin^2 x(\csc^2 x - 1)$	30. $\cot a \sin a$

Verify each identity is true using the identities above, and transforming only the left side of each equation.

31. $(1+\cos\theta)(1-\cos\theta) = \sin^2\theta$	32. $\frac{\sec\theta - 1}{1-\cos\theta} = \sec\theta$

VI. Calculator Proficiency.

During our AP Calculus course, about 2/3 of our work will be done without the use of a calculator (2/3 of the AP test will also be calculator-less). However, there are certain skills carried over from previous classes that are critical to your calculator work in AP Calculus.

The instructional links here are designed for a TI-84/TI-83 model calculator. If you are using a TI-Nspire, TI-89, TI-86, or a Casio graphing calculator, you will have some additional learning to do outside of this document.

The links provided for each section will give you a basic introduction to a function that you will be asked to perform frequently this year.

Using Store and Recall

http://online.math.uh.edu/GraphCalc/STORE-Example/STORE-Example.html

33. Use the store feature to evaluate the following expressions at the given values.

(a) $f(x)=3x^{\frac{1}{2}}-4e^{x+1}$ $f(3.57)$	(b) $g(x)=\dfrac{x^2-8x+13}{2\sin(x-7)-3\cos(2x-8)}$ $g(e^{1.5})$

Finding Zeros of a Function from the graphing screen.

http://online.math.uh.edu/GraphCalc/Zero/Zero.html

34. Use zero option in the calc menu to find all zeros for the following functions.

(a) $h(x)=x^{1.5}-4$	(b) $j(x)=2x^2-5x+3$
(c) $k(x)=2\sin(x-\frac{\pi}{2}),0\le x\le 2\pi$	(d) $m(x)=x^2e^x-x$

Finding the Intersection of two Functions from the graphing screen.

http://online.math.uh.edu/GraphCalc/Intersect/Intersect.html

35. Solve each system of equations by finding the intersection using your calculator.

(a) $y=2e^{x-2}$ $y=-x^2+4$	(b) $y=-\sqrt{25-x^2}$ $y=-2$
(c) $y=\dfrac{2}{3-x}$ $y=x^3-2x$	(d) $y=\sin(3x)$ $y=3-e^x$